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New methods from mathematics and mathematical logic were employed to generalize the event algebra of probability theory while retaining the numerical, algebraic structure of its probability functions. One form of generalization developed consisted of extending the event space of standard probability theory to include events corresponding to counterfactuals. Another form of generalization researched extended the probability concept to arbitrary lattices (with maximal and minimal elements) and characterized how the probability concept structurally restricted a lattice to which it is applied. Because lattices can be viewed as propositional logics (either classical or non-classical) or as event spaces (either boolean or non-boolean), this is equivalent to understanding how the probability concept restricted the underlying logic or event space on which it was defined. The generalizations were applied to the empirical literature about probability judgments and to game theory. For probability judgments, a new foundation was given for the most prominent theory of probability judgments in psychology, ``Support Theory,'' and a new method was developed for the analysis of several puzzling empirical phenomena involving probability judgments. For game theory, a new approach to deterrence based on counterfactual reasoning was developed.

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Foundational Issues in the Measurement of Belief and Uncertainty

Grant Award Number FA9550-08-1-0389
Dr. Jay Myung

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University of California, Irvine
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0. Preamble & Introduction New methods from mathematics and mathematical logic were employed to generalize the event algebra of probability theory while retaining the numerical, algebraic structure of its probability functions. Some of the generalizations resulted from "Dutch Book" arguments. Economists, statisticians, and philosophers employed Dutch Book arguments to establish a rational basis for probability theory. The key ideas of these arguments transferred to the generalizations developed in this grant, and, because of this, the generalizations were similarly rational. A second kind of generalization developed consisted of extending the event space of standard probability theory to include events corresponding to counterfactuals. This type of generalization was also rationally based. A third form of generalization researched extended the probability concept to arbitrary lattices (with maximal and minimal elements) and characterized how the probability concept structurally restricted a lattice to which it was applied. Because lattices could be viewed as propositional logics (either classical or non-classical) or as event spaces (either boolean or non-boolean), this was equivalent to understanding how the probability concept restricted its underlying logic or event space. The generalizations were then applied to the empirical literature about probability judgments and to game theory. For probability judgments, a new foundation was given for the most prominent theory of probability judgments in psychology, "Support Theory," and a new method was developed for the analysis of several puzzling empirical phenomena involving probability judgments. For game theory, a new approach to deterrence based on counterfactual reasoning was developed. In addition, during the course of the grant it was discovered that one of the new event spaces had an interpretation as the logical structure of knowledge, and because of this, additional investigations were undertaken to apply this to the use of knowledge in games and to the self-organization of systems of shared knowledge among people.

1. Scientific Objectives of the Research

The research proposed in the grant consisted of the following objectives:

- Extend the probability concept to non-boolean event spaces and non-classical logics.
- Derive these event spaces and logics through rationality arguments similar to Dutch Book arguments for standard probability theory.
- Apply at least one of the event spaces and logics in to provide a new foundation for the large empirical literature on human probability judgments.
- Provide a probability theory based on rationality considerations that incorporates counterfactual propositions and apply it to models of deterrence.

All of these objectives were met. While researching them, we made discoveries that led us develop additional objectives that strengthened the research concerning original objectives as well help in future theoretical and empirical research by us and others interested in the grant's topics. These additional objectives are:

- Axiomatize the logical structure of knowledge and describe its non-boolean event space.
- Apply the axiomatization of knowledge to issues involving knowledge in strategic game theory.
- Develop and model concepts of shared knowledge and apply them to normative and psychological issues in cooperative game theory.
- Incorporate emotion and other kinds of contexts that impact decisions into decision theory through use of non-boolean event spaces that have been developed as part of this grant's research.
- Develop new theory and experimental methods for measuring the impact of context on probabilistic judgments.

Much progress has been made on all of these additional objectives.

2. Technical Approach

The technical approaches to our research consisted of the following: (1) the use of theoretical mathematics, mostly in the theory of lattices, to produce new, applicable mathematical results; (2) the use of mathematical logic, mostly involving intuitionistic logical approaches used in the foundations of mathematics, to develop a new probability theory based on a new kind of event space and use these approaches to provide a new foundation for decision theory; (3) the use of philosophical logic, particularly counterfactual and modal logics, to develop approaches to higher-order reasoning in game theory and the logical structure and use of knowledge in game theory; (4) the use of evolutionary game-theoretic algorithms and simulations to investigate the role of learning and shared knowledge in games; and (5) the development and use of psychophysical techniques for deciding whether measurements of psychological subjective intensities of different stimuli (e.g., probability judgments of uncertainty) were on the same or different scales.

3. Progress Mad & Results Obtained

Objectives generalizing the probability concept and the Dutch Book Argument Modern probability theory was based on a finitely additive (or, alternatively, a σ-additive) function on an event space that was a boolean algebra. Various empirical studies on human decisions called into question the adequacy of using modern probability theory as a model for human probability judgments that were part of decision situations involving uncertainty. The usual approach in psychological and economic modeling was to change the algebraic properties of the probability function so that various forms of non-additivity results. These forms of non-additivity violated conditions that are at the foundation for the rational justifications of modern probability theory. Our proposed approach was to change the algebraic properties of the event space, while preserving the key rationality assumptions of modern probability theory. We accomplished this objective by basing probability theory on a collection of open sets from a topology. Modern probability theory then becomes the degenerate case where the topology is such that every open set is also a closed set. In such a degenerate case, disjoint open sets cannot have a common boundary point.

Generalizing the probability concept. Part of our proposed research was to de-

velop the underlying mathematics for the new probability theory using such topological event spaces. A truncated part of our work on this topic was published in Narens (2011). The topic, as originally envisioned, turned out to be very complicated: A substantial number theorems from the lattice theory literature of mathematics needed to be cited as part of our theoretical development of the new probability theory. But these theorems were scattered over more than two dozen publications, and integrating them into our newly developed theory meant that besides them, many definitions of concepts of general lattice theory had to be given as additional supporting theorems that we developed in order to link ideas together. As a result, we decided that this part of the research was better presented as a book rather than a series of truncated articles, like Narens (2011). In the book manuscript, *Probabilistic Lattices with Applications to the Behavioral Sciences* (Narens, 2012a), the mathematical development was able to be organized so that the integration of the literature with our new results flowed together, while providing all the necessary background material needed for a precise understanding of the results. The latter was required, because almost no scientists and only a few mathematicians were familiar with basic, general lattice theory, in contrast to, say, vector spaces, group theory, or probability theory. Because of this, the production of the book would have a bigger impact than a series of articles, since it also taught the relevant lattice theory necessary for understanding the research. The theoretical, mathematical material for the book has been drafted. What was needed for publication was polishing and a small amount of text linking material between chapters. A graduate course cross-listed in the Departments of Cognitive Sciences, Economics, and Logic and the Philosophy of Science has been scheduled for Winter Quarter, 2013, UC Irvine, with the book as the main text. The use of the book in this course will help in additional polishing of the material and the elimination of technical errors. A publication date before July 1, 2013 is realistic. Both PIs have published many, single authored research books, and in our experience, the time frame described above is easily achievable.

Dutch Book Argument. In the 1970's, Bruno de Finetti founded the theory of subjective probability on coherence. Coherent degrees of belief was impervious to a "Dutch Book," while incoherent degrees of belief was subject to a Dutch Book. A Dutch Book consisted in a finite number of wagers, all of which were favorable according to the degrees of belief in question, and such that the net outcome was negative to the wagerer no matter how the state of the world that was the object of the wagers fell out. In the classical set-up coherence was defined in

terms of a boolean algebra of events. We were able to relax this assumption to general lattices and still showed that was equivalent to the existence of a specific generalized form of the classical probability function on the lattice that allows for "infinitestimal probabilities." This result gave rise to more general possibilities for the structure of subjective probability. This research extended previous work by Narens and others concerning the existence of quantitative probability representations based on rationality assumptions about qualitative orderings of events. Because the proof used ideas that are unfamiliar to most mathematicians and scientists, it was presented in the book *Probabilistic Lattices with Applications to the Behavioral Sciences* instead of an independent article, because the book develop all the relevant background concepts and theorems.

Objective to apply at least one of the event spaces and logics in to provide a new foundation for the large empirical literature on human probability judgments The current dominate theory in psychology of subjective estimates of probability is called "Support Theory". It was formally introduced in the 1990's by Amos Tversky. In 2007, Narens presented a radically different foundation based on an event space of open sets. Using mathematical research developed in this grant, Narens (2009) extended this foundation by giving a more detailed account of the role of the complementation operation in the lattice of open sets and how it was related to the concepts of recall and recognition memory in cognitive psychology. (In the lattice of open sets, the open complement of an open set A is the interior of the set-theoretic complement of A. This complement was different from set-theoretic complementation.)

Objective involving Rationality, Counterfactual Propositions, and Models of Deterrence We were able to develop a new game-theoretic approach of deterrence using rationality considerations and counterfactual logic. (The following is an example illustrating the importance of counterfactual thinking in decision making and policy analysis: We agree that (1) *If Oswald didn't kill Kennedy, someone else did,* but deny that (2) *If Oswald hadn't killed Kennedy someone else would have.* Confusing (1) and (2) in policy analysis would be disastrous.) This approach, described in Skyrms (2012a), was based on the idea that rational decision depended on the evaluation of conditionals—that is, of what would happen if actions, strategies, or policies were implemented. More generally (and more realistically) decision makers were required to assess what the probabilities of alternative consequences would be if specific actions were taken. Evaluation of such

conditionals about probable consequences would be an especially complex problem in contexts of strategic interaction in which chances of an outcome for person A depended not only on A's actions, but also on B's and C's response to A's actions, which may in turn depended on their beliefs about A's probable response to their actions. The logic of such hypothetical reasoning about chances of effects of actions was fundamental to analysis of strategic interaction.

Objective to axiomatize the logical structure of knowledge and describe its non-boolean event space We developed a minimal theory of knowledge. The theory was not intended as a complete theory for rational knowledge as used in philosophical and economic treatments of the subject, but a theory that could be extended to such a theory. The theory was designed for scientific application, and it left out much many issues of philosophic importance. We showed that if this theory held then we could form an event space corresponding to the extension of classical propositional logic that included "P knows" as an additional propositional operator. For each proposition a in this extension, there was a corresponding event A consisting of the states of the world in which P knows a. We showed that this event space was the kind of topological event space described above that we developed for our new probability concept.

The primitive concept of modern probability theory was the probability that a proposition a is true. With the just-mentioned research this could be changed to a probability theory with a different primitive concept: the probability that person P knows proposition a. We believe that this change could have important applications in game theory.

Objective to apply the axiomatization of knowledge to issues involving knowledge in strategic game theory During the last year of the grant, an economist Willemien Kets joined our project as a Post Doctoral Researcher to investigate the role of knowledge in game and decision theory. The standard framework in economics for analyzing games with incomplete information modeled players as if they have an infinite depth of reasoning. For the kind of applications that we intended to apply of our event knowledge space to—in particular, applications to psychology and behavioral economics—we needed an appropriate economic generalization to situations that had a finite depth of reasoning. This was accomplished in Kets (2012a).

Objective to develop and model concepts of shared knowledge and apply them to normative and psychological issues in cooperative game theory We have done this from both mathematical and simulation perspectives. This objective generated three kinds of sub-projects.

1. Standard models of multi-agent modal logic did not capture the fact that information was often ambiguous, and could be interpreted in different ways by different agents. Kets & Halpern (2012) developed a framework that for modeling this, by using different semantics to capture different kinds of assumptions about the agents' beliefs regarding whether or not there is ambiguity. Kets & Halpern (2012) investigated the impact of ambiguity on a seminal result in economics by Aumann that said agents with a common prior cannot agree to disagree. Aumann's result does not hold if agents do not have a common prior. Kets and Halpern showed that it also does not hold in the presence of ambiguity. They then investigated the properties of the trade-off between assuming a common interpretation (i.e., no ambiguity) and a common prior (i.e., shared initial beliefs).

2. Narens, Jameson, Komarova, & Tauber (2012) investigated how shared knowledge involving concepts aroused for color categories in populations of simulated agents. The method used concepts from psychology concerning perceptual discrimination and learning and concepts from game theory concerning evolutionary processes. A language game where agents assigned names to color patches was played repeatedly by members of a population. The dynamics employed made minimal assumptions about agents' perceptions and learning processes. Through various simulations it was shown that under different kinds of reasonable conditions involving outcomes of individual games, the dynamics pushed populations to stationary equilibria, which was interpreted as achieving shared population meaning systems. Optimal population agreement for meaning was characterized through a mathematical formula, and the simulations presented revealed that for a some of situations, near optimality was achieved. An important goal of this research was to demonstrate how the networking of individual agents affected the evolution of the shared knowledge of the population. This led to the conclusion that various modeling processes used by psychologists and other researchers to achieve shared meaning in a population were highly flawed for accounting for how this was achieved by human populations, because the equilibrium of shared meaning was largely a result of individuals randomly interacting with other individuals, and that such equilibria were not obtained or were different when a reasonable amount of non-random interaction was included by the used of a net-

work structure. (Simulations of varying size up to 10,000 agents were employed, and various network structures, including "small world networks" from sociology were considered.)

3. Signaling is a key ingredient in the evolution of teamwork not only in the human world but also in the animal world, even in micro-organisms. Communication and coordination of action are different aspects of the flow of information, and are both effected by signals, and both are involved in how a population of agents (e.g., people or artificial agents) could learn and share knowledge through signaling games. Skyrms (2010a), in his book *Signals: Evolution, Learning, and Information,* presented a new and comprehensive account of signals. This book (as well as several articles by Skyrms and colleagues listed below under "Publications") developed new theories of signaling games as well as new approaches to information, evolution, and learning. Skyrms employed these to investigate how meaning, communication, and networks could evolve in systems that originally had only random meaning for individuals, random methods of communication, and no initial network structure. He also showed how signaling games themselves evolve, and introduced a new model of learning with invention. In his models, the juxtaposition of atomic signals led to complex signals, as the natural product of gradual process. Also, in his models, signals operated in networks of senders and receivers with the transmitted information being processed in various ways.

Objective to incorporate emotion and other kinds of contexts that impact decisions into decision theory through use of non-boolean event spaces that have been developed as part of this grant's research Narens (2012b) expanded the standard rational decision model used in economics and science, SEU, to incorporate emotion, bias, and other modes that could influence a decision maker's judgment of utilities of lotteries. In multimode utility theory (MUT), judgements were based on subjective interpretations that often varied with the mode, resulting in the same event having multiple interpretations and the decision maker making mode dependent judgments. In MUT, an event's multiple interpretations were modeled in a manner so that they were related semantically. The semantic relationship was characterized through topological and algebraic means, using non-boolean event space consisting of open sets from a topology that was developed in other parts of this grant. This was done in a way that allowed for systems of probabilities and decision making for lotteries that were arguably *subjectively coherent.* They were not, however, necessarily *behaviorally coherent,* e.g., the Dutch book crite-

ria for rationality developed in other parts of this grant could fail. The conflict between the simultaneous holding of subjective coherence and the failure of behavioral coherence posed interesting rationality questions that were discussed in Narens (2012b), e.g., Which coherence concepts should "rationality" be based on? Was SEU's primitive concepts too impoverished to provide a reasonably realistic theory for rational human decision making?

MUT provided alternative methods for modeling various phenomena in the behavioral economic and psychological literatures. In particular, Narens (2012b) used it to model decision making for lotteries with catastrophic events or catastrophic outcomes as well as to model situations where emotions such as fear and hope influenced decisions.

Objective to develop new theory and experimental methods for measuring the impact of context on probabilistic judgments In previous research Narens developed a theoretical model for deciding if subjective judgments about objects from a domain were made on the same or on different scales. For subjective judgments of probability, it was important to know if subjective probabilities for various events, e.g., the subjective probability of a devastating earthquake occurring in Los Angeles, the subjective probability of winning a particular lottery, etc., were measured by the same or different probability functions. Because this grant did not plan for experimental research, we could not carryout experimental work on this topic. However, another AFOSR grant to Duncan Luce, PI, (FA9550-08-1-0468) planned experimental research on the related problem using judgments of subjective intensity instead of subjective probabilities. Narens' theoretical model applied to both kinds of judgments. Because of this, we collaborated with Luce's team to test empirically Narens' theoretical model for psychophysical intensity judgments. The articles Luce, Steingrimsson, & Narens (2011) and Steingrimsson, Luce, & Narens (2012) showed that empirical test held for psychophysical stimuli, that is, they showed that the loudness of different tones were judged on the same scale (the 2011 article), and they showed that the brightness of different colors were judged on the same scale (the 2012 article).

4. Significance of Results & Impact on Science Standard probability theory has had enormous success and wide application throughout science. Thus any significant change or improvement of probability theory is likely to have signifi-

cant potential impact on science. An important part of the foundation of standard probability was its use of a boolean algebra of events as its event space. For some scientific situations, this turned out to be the wrong structure for the event space, for example, in the 1930's physics discovered that a different kind of event space—closed subspaces of a Hilbert space—was needed to model the kind of phenomena encountered in quantum mechanics. The research conducted for this grant suggested that something similar is needed for human decision theory: For many kinds of human decisions uncertainty, the event space is better modeled by events from a topological space instead of a boolean algebra of events.

Boolean events either happened or did not happen. In addition to this possibility, topological events could also have ambiguous happened (e.g., the happening or not happening of an event depended on the context in which the event was presented), vaguely have happened (e.g., there was some information concerning the happening of the event and no information concerning the event not happening), or indefinitely have happened, (if it happened it could be verified that it happened; if it didn't happen, it may be impossible to verify it didn't happen). This made topological modeling of events more realistic in the modeling of human decision making, while—as shown in this grant—providing for an alternative, rigorous, mathematical basis for its probability theory. Our research also showed that topological modeling could be used to provided new psychological insights into cognitive theories of decision making (Narens, 2009) as well as new kinds of models for normative, economic decision making (Narens, 2012). Other work of ours for this grant showed that counterfactual logic provided a rigorous foundation for modeling important game-theoretic situations involving deterrence (Skyrms 2012a).

Thus our research suggested that the event structure of many important decision theoretic situations have been mis-modeled in the literature. Our research also suggested that this can be corrected by new forms mathematical modeling for the event structure of probability theory. Also a well-known argument for rationality of standard probability theory was extended to argue for the rationality of the new approaches. In addition we applied these approaches (and developed additional ones) for issues involving shared knowledge in game theory, and applied them to some long-standing issues in game theory (Skyrms, 2010a; Narens, Jameson, Komarova, Tauber, 2012; Skyrms, 2009d; Kets, 2012; Kets & Halpern, 2012).

5. Publications

Books

Narens, L. (2012a). *Probabilistic lattices with Applications to the Behavioral Sciences.* Manuscript

Skyrms, B. (2010a). *Signals: Evolution, Learning, and Information.* Oxford University Press.

Articles and Chapters

Narens, L. (2012b). Multimode Utility Theory. Submitted.

Kets, W. (2012). Bounded Reasoning and Higher-Order Uncertainty. Submitted.

Skyrms, B. (2012a). The Core Theory of Conditionals. In press, *Synthese*.

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Narens, L. (2009). A foundation for support theory based on a non-Boolean event space. *Journal of Mathematical Psychology*, 2009, 53, 399–407.

Skyrms, B. (2009a). Common Knowledge, the Cooperative Principle and Adaptive Dynamics. In M. Tomasello, Ed.,*Why We Cooperate.* Cambridge, Mass.: The MIT Press.

Skyrms, B. (2009b). Evolution of Signaling Systems with Multiple Senders and Receivers. *Philosophical Transactions of the Royal Society B., 364,* 771–779.

Skyrms, B. (2009c) Groups and Networks: Their Role in the Evolution of Cooperation. In S. Levin, Ed., *Games, Groups and the Global Good.* Berlin: Springer, 105–114.

Skyrms, B. (2009d). Evolution and the Social Contract. In *The Tanner Lectures on Human Values, 28.* Salt Lake City: University of Utah Press, 47–63.